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New linear approximation method applied to crude operations scheduling

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Resumen

Este artículo aborda el desarrollo de un nuevo método de aproximación lineal para el producto de dos variables continuas, aplicado a la optimización de la programación de operaciones de crudo en una refinería abastecida por barcos. Un reto clave en este problema de programación de operaciones es la gestión del almacenamiento de crudos en tanques. Dado que la capacidad de almacenamiento es limitada y que existen varios tipos de crudos con diferentes composiciones, es necesario almacenar mezclas de crudos en los tanques. Esta característica introduce restricciones no lineales y no convexas, lo que complica la resolución de los modelos de programación matemática. Con el fin de superar este problema, hemos desarrollado una estrategia basada en aproximación lineal por tramos utilizando planos, que permite tratar eficientemente las restricciones no lineales asociadas a la mezcla de crudos en tanques.

Palabras clave: Modelado y toma de decisiones en sistemas complejos, Estrategias eficientes para sistemas complejos a gran escala, Planificación y control de la producción, Optimización y control de sistemas de redes a gran escala, Sistemas logísticos complejos.

Abstract

This article focuses on the development of a new linear approximation method for the product of two continuous variables, which is applied to the optimization of crude oil operations scheduling in a refinery supplied by ships. A key challenge in this scheduling problem is the management of crude oil storage in tanks. Since the storage capacity is limited and there are different types of crude oil with different compositions, it is necessary to store mixtures of crude oils in the tanks. This feature introduces nonlinear and non-convex constraints, which complicate the solution of the mathematical programming models. To overcome this problem, we have developed a strategy based on piecewise linear approximation using planes, which efficiently handles the nonlinear constraints associated with crude oil blending in tanks.

Keywords: Modelling and decision making in complex systems, Efficient strategies for large scale complex systems, Production planning and control, Optimization and control of large-scale network systems, Complex logistic systems.

1. Introduction

In this paper, we develop a new method that allows the linear approximation of the product of two continuous variables. The motivation for this development stems from the problem of blending crude oils in tanks, which is present in crude oil operations scheduling.

In the mathematical modeling of the crude oil operations scheduling problem, a constraint must be included that equates the composition of the stored crude oil blend with the composition of the output blend for each storage tank. This constraint is nonlinear and non-convex because it includes two bilinear terms, each of which is the product of two continuous variables related to the inventory level and the volume trans-

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ferred from the tanks to the distillation units.

The method presented in this work consists of a piecewise linear approximation using planes. To evaluate the performance of this method, a crude oil operations scheduling problem based on a case study from the literature was solved and compared with other methods: piecewise McCormick approximation and DICOPT.

Next, we describe the crude oil operations scheduling optimization problem in a refinery supplied by ships to provide the reader with the context of the problem that motivated the development of the linear approximation method. We analyze a system composed of a marine terminal, and an oil storage and processing unit connected by a pipeline. The terminal serves as the location for unloading the crude oil transported by the ships. We consider a single dock terminal, which allows the unloading of one ship at a time.

Concerning the storage and processing section, it is divided into two areas: the storage tank area and the crude distillation unit area. The first is connected to the marine terminal by a pipeline and, as the name suggests, consists of tanks for storing crude oil received from the terminal. Most refineries have two types of tanks: storage tanks, which receive and store crude oil from ships, and charging tanks, used for creating blends to feed distillation units, meeting certain quality specifications. Due to the traditional operation of refineries utilizing both types of tanks, a wide variety of articles addressing the optimization of crude oil operations scheduling in such refineries have been published in recent decades (Lee et al. (1996), Mouret et al. (2009), Castro and Grossmann (2014), Yang et al. (2020)). However, some refineries opt to eliminate charging tanks to save space and reduce immobilized capital. Instead, they implement online mixing in the pipelines feeding the crude distillation units (CDUs) using a suitable control system. While researchers have studied this case, there is a smaller number of published works focusing on this type of refinery (Cerdá et al. (2015), García-Verdier et al. (2022)).

An important characteristic in both cases is that the concentration of crude oil in the outflow of a tank must be equal to the concentration inside the tank. This behavior is represented by a set of nonlinear non-convex constraints that give rise to mixed-integer nonlinear programming (MINLP) models that are difficult to solve. In the literature, we can find works proposing different strategies to address this problem. For example, in de Assis et al. (2017), the authors propose a two-step MILP-NLP decomposition algorithm, where the mixed-integer linear programming (MILP) model is obtained by replacing each side of the nonlinear constraint with piecewise McCormick envelopes. Also, the work presented in Castro and Grossmann (2014) introduces a two-step MILP-NLP algorithm, where the bilinear blending constraints are relaxed using multiparametric disaggregation. This technique involves discretizing one of the variables of the bilinear term over a set of powers. In Garcia-Verdier et al. (2024), the authors propose a two-step MILP-NLP algorithm, where the approximate MILP formulation is obtained by replacing the nonlinear constraint with linear constraints, which determine that a tank maintains the initial crude concentration until the moment it receives crude oil from a ship. It is worth noting that in none of the cases is a procedure proposed in case the nonlinear programming (NLP) solution is infeasible.

Continuing with the description of the system, the tanks are connected to the crude distillation unit area through a piping system (mixing pipelines), where the final mixtures of crudes take place to achieve the desired flows and properties required by the different crude distillation units (CDUs). All operations are subjected to multiple rules and constraints, among them, the arrival over time of different types and amounts of crude, and the fulfillment of the company production plan. Figure 1 shows a schematic of the refinery under study, which has only storage tanks.

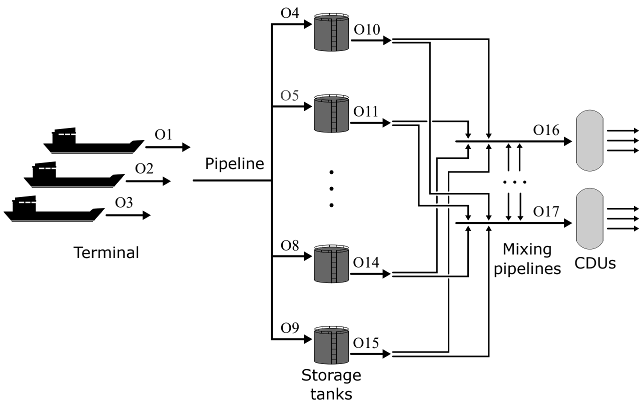

Figure 1: Schematic of refinery.

The aim of this work is to develop a strategy to address the nonlinear non-convex constraints generated by blending crude oils in tanks. First, an approximate MILP model is obtained based on piecewise approximation using planes. From the solution of this MILP, we fix the binary variables of the original MINLP and solve the resulting NLP. Finally, if a feasible solution is not obtained, a cut is added to the MILP model, and the procedure is repeated.

The rest of the paper is structured as follows. The proposed strategy is described in Section 2. The solution procedure for the crude oil operations scheduling MINLP model is discussed in Section 3. Next, a problem instance and computational results are reported in Section 4. Finally, conclusions are given in Section 5.

2. Proposed strategy

As mentioned earlier, in crude oil operations scheduling, we must consider the blending of crude oils in tanks, which results in a nonlinear constraint involving products of continuous variables. Therefore, this section presents the strategy developed to linearly approximate the product of two non-negative continuous variables. For the sake of clarity and to avoid complicated nomenclature, let us assume that we want to approximate the product of the variables $x1$ and $x2$. Note that this approximation can also be applied to the product of indexed variables.

First, the domain of each variable is partitioned into a certain number of intervals. Additionally, each quadrilateral formed by the intersection of intervals is divided into two triangles. For this, the diagonal is drawn from the lower left vertex to the upper right vertex in each quadrilateral. Then, from the vertices of each triangle, a plane is defined that will approximate the product of these variables in each interval. It should be noted that depending on the location of the point to be evaluated, one plane or another will be used. If the point is located above the diagonal, the equation formed by the vertices at the lower left, upper left, and upper right is used. If the point is below the diagonal, the equation formed by the vertices at the lower left, lower right, and upper right is used (Figure 2).

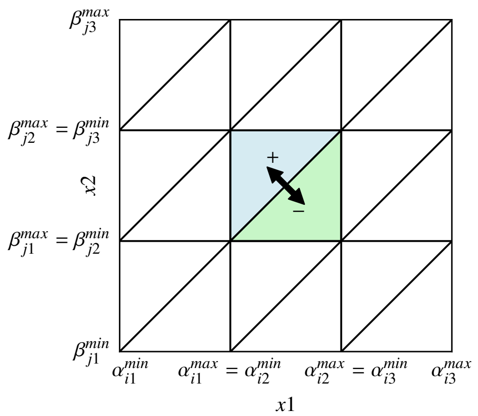

Figure 2: Piecewise approximation by planes.

Based on the above premise, the following set of variables and constraints is defined.

Each variable is disaggregated into two variables: "upper" and "lower". The "upper" variable ($x1_{i,j}^u$ for $x1$, and $x2_{i,j}^u$ for $x2$) is related to the upper plane, and the "lower" variable ($x1_{i,j}^l$ for $x1$, and $x2_{i,j}^l$ for $x2$) to the lower plane. Constraints (1) and (2) represent the disaggregation of variables $x1$ and $x2$, respectively. Set I is the set of intervals for $x1$, and set J is the set of intervals for $x2$.

$$x1 = \sum_{i \in I} \sum_{j \in J} x1_{i,j}^u + x1_{i,j}^l \tag{1}$$

$$x2 = \sum_{i \in I} \sum_{j \in J} x2_{i,j}^u + x2_{i,j}^l \tag{2}$$

Two binary variables ($w_{i,j}^u$ and $w_{i,j}^l$) are defined to determine to which quadrilateral the point to be evaluated belongs, and to which plane (upper or lower). For example, if $x1$ is in interval i, $x2$ in interval j, and the point $(x1, x2)$ is above the diagonal; then $w_{i,j}^u$ will be equal to 1. The constraints (3)-(6) establish the upper and lower bounds of each disaggregated variable in each quadrilateral. The parameters α_i^{min} and α_i^{max} represent the minimum and maximum values, respectively, in interval i for the variables $x1_{i,j}^u$ and $x1_{i,j}^l$. Similarly, the parameters β_j^{min} and β_j^{max} represent the minimum and maximum values, respectively, in interval j for the variables $x2_{i,j}^u$ and $x2_{i,j}^l$.

$$\alpha_i^{min} * w_{i,j}^u \leq x1_{i,j}^u \leq \alpha_i^{max} * w_{i,j}^u \quad \forall i \in I, j \in J \tag{3}$$

$$\alpha_i^{min} * w_{i,j}^l \leq x1_{i,j}^l \leq \alpha_i^{max} * w_{i,j}^l \quad \forall i \in I, j \in J \tag{4}$$

$$\beta_j^{min} * w_{i,j}^u \leq x2_{i,j}^u \leq \beta_j^{max} * w_{i,j}^u \quad \forall i \in I, j \in J \tag{5}$$

$$\beta_j^{min} * w_{i,j}^l \leq x2_{i,j}^l \leq \beta_j^{max} * w_{i,j}^l \quad \forall i \in I, j \in J \tag{6}$$

The variable z, representing the value of the product of $x1$ and $x2$, is also disaggregated (7).

$$z = \sum_{i \in I} \sum_{j \in J} z_{i,j}^u + z_{i,j}^l \tag{7}$$

The disaggregated variables are calculated by (8) and (9). Note that these constraints correspond to the equations of the upper and lower planes for each quadrilateral, respectively. Additionally, if a plane is not selected, the value of the disaggregated variable associated with it will be zero.

$$z_{i,j}^u = \beta_j^{max} * x1_{i,j}^u + \alpha_i^{min} * x2_{i,j}^u - \alpha_i^{min} * \beta_j^{max} * w_{i,j}^u \quad \forall i \in I, j \in J \tag{8}$$

$$z_{i,j}^l = \beta_j^{min} * x1_{i,j}^l + \alpha_i^{max} * x2_{i,j}^l - \alpha_i^{max} * \beta_j^{min} * w_{i,j}^u \quad \forall i \in I, j \in J \tag{9}$$

Constraint (10) states that only one plane can be selected.

$$\sum_{i \in I} \sum_{j \in J} w_{i,j}^u + w_{i,j}^l = 1 \tag{10}$$

Depending on whether the distance from the point to the diagonal is positive (11) or negative (12), the binary variable $w_{i,j}^u$ or the binary variable $w_{i,j}^l$ can be activated, respectively.

$$(\alpha_i^{max} - \alpha_i^{min}) * x2_{i,j}^u - (\beta_j^{max} - \beta_j^{min}) * x1_{i,j}^u \geq (\alpha_i^{min} * \beta_j^{max} - \alpha_i^{max} * \beta_j^{min}) * w_{i,j}^u \quad \forall i \in I, j \in J \tag{11}$$

$$(\alpha_i^{max} - \alpha_i^{min}) * x2_{i,j}^l - (\beta_j^{max} - \beta_j^{min}) * x1_{i,j}^l \leq (\alpha_i^{min} * \beta_j^{max} - \alpha_i^{max} * \beta_j^{min}) * w_{i,j}^l \quad \forall i \in I, j \in J \tag{12}$$

3. MINLP solution procedure

As mentioned in the introduction, one of the aim of this work is to solve the crude oil operations scheduling problem of a refinery, considering blending in tanks. Below are some of the constraints that must be considered when formulating the mathematical programming model for this process:

- Vessel operation
- Vessel to tank allocation
- Tank to unit allocation

- Mass balance (inventory level)
- Calculation of key property values
- Demurrage and tardiness
- Crude oil concentration at the outlet of a tank (nonlinear)

Due to the page limit, the detailed MINLP model based on continuous time using slots is omitted. However, we present the nonlinear constraint which states that if a tank is being unloaded, then the crude oil concentration in the outflow must be equal to the concentration inside the tank. Below, only the notation of sets and variables involved in this nonlinear constraint is provided.

3.1. Notation

3.1.1. Sets

- C = types of crude oils
- N = time slots
- O = operations
- OPO = pair of operations (o, o'), where operation o can activate operation o'
- R = resources
- $RS \subset R$ = tanks
- $OUTR_r$ = pair (o, r), o is an output operation of r

3.1.2. Continuous variables

- $IIN_{r,n}$ = inventory level in tank r after receiving a load during slot n
- $IINC_{r,n,c}$ = inventory level of crude c in tank r after receiving a load during slot n
- $VP_{o,n,o',n'}$ = volume transferred by operation o during slot n to operation o' assigned to slot n'
- $VPC_{o,n,o',n',c}$ = volume transferred of crude c by operation o during n to operation o' assigned to n'

3.1.3. Binary variables

- $X_{o,n,o',n'}$ = indicates if (o, n) produces (o', n')

Given the nonconvex nature of the nonlinear constraint (13), it is essential to explore strategies to effectively address this challenge.

$$IIN_{r,n} * VPC_{o,n,o',n',c} = IINC_{r,n,c} * VP_{o,n,o',n'} \quad o, o' \in OPO,\ o \in OUTR_r,\ r \in RS,\ n, n' \in N,\ c \in C \tag{13}$$

In this paper we implement the following procedure to solve the MINLP model. Initially, we apply the approximation presented in section 2 to each term of (13), and equate the variables representing each term. Substituting the nonlinear equation with the corresponding set of constraints, we obtain and solve a MILP model. Subsequently, the binary variables in the original MINLP are fixed based on the solution found for the MILP, and the resulting NLP model is solved.

If the solution is not feasible, we add the following "no good" constraint (14) to the MILP model and solve it again, forcing at least one of the variables to change value. This procedure is repeated until a feasible solution for the NLP is obtained or a certain number of iterations is exceeded.

$$\sum_{(o,n,o',n'):\hat{X}_{o,n,o',n'}=0} X_{o,n,o',n'} + \sum_{(o,n,o',n'):\hat{X}_{o,n,o',n'}=1} (1 - X_{o,n,o',n'}) \geq 1 \tag{14}$$

4. Results

To assess the performance of the proposed strategy, we conducted a case study based on problem 2 from Mouret et al. (2009), with a configuration corresponding to Figure 1. Table 1 summarizes the problem data. Additionally, the suggested number of time slots is eight. We solved the problem using GAMS 43.2 software, Gurobi 9.5.2 for MILPs, and CONOPT 4.29 for NLPs on a computer equipped with an Intel Core i9-13900K 3.00 GHz processor and 128 GB of RAM.

The example has been solved in three ways: by applying piecewise approximation by planes (PAP), implementing piecewise McCormick approximation (MCC), and using the DICOPT solver. The objective function aims to minimize vessel demurrage and tardiness costs while maximizing the profit from processed crude.

Table 2 shows the objective function values and computation times for each case. It also details the number of continuous and binary variables, as well as the number of constraints for the MILP models based on PAP and MCC, and for the MINLP model solved with DICOPT. The number of continuous variables and constraints for the NLP models associated with PAP and MCC equals that of the MINLP model. It should be noted that both strategies, PAP and MCC, used a single interval. Also, from Table 2, we can see that the PAP and DICOPT solutions yielded a profit of \$17.500.000, whereas the MCC solution resulted in a lower profit of \$16.900.000. In terms of model size, i.e. number of variables and constraints, we can notice that the PAP model is one order of magnitude larger than the other two. However, it also has the shortest solution time.

Figure 3 depicts a Gantt chart of the PAP solution. As shown, ships start and finish their unloading operations on schedule. Specifically, the unloading of ship 1 leads to the loading of tanks 1 and 4 (operations O4 and O7, respectively). Ship 2 unloads into tank 5, and ship 3 unloads into tank 6. Tanks 2 and 3 receive no cargo. Finally, it should be noted that the CDUs operate continuously (operations O16 and O17).

Table 1: Problem data.

Vessel	Arrival time (day)	Cargo	Volume (Mbbl)	Mixture	Specification	Demand (Mbbl)
1	0	100% A	1000	1	[0,01, 0,02]	1500
2	4	100% B	1000	2	[0,025, 0,035]	900
3	10	100% C	1000	3	[0,04, 0,048]	800
Tank	**Capacity (Mbbl)**	**Initial composition**	**Amount (Mbbl)**	**Crude**	**Key property**	**Gross margin ($/Mbbl)**
1	[0, 1000]	100% A	200	A	0.01	1
2	[0, 1000]	100% B	500	B	0.03	3
3	[0, 1000]	100% C	700	C	0.05	5
4	[0, 1000]	100% D	300	D	0.0167	1.67
5	[0, 1000]	100% E	300	E	0.03	3
6	[0, 1000]	100% F	300	F	0.0433	4.33
Scheduling horizon			15 days	Unloading and transfer flowrate (Mbbl/day)	[0, 500]	Distillation flowrate [50, 500]

Table 2: Solutions and model statistics.

Strategy	Profit ($)	Continuous variables	Binary variables	Constraints	Time (s)
PAP	17.500.000	100.879	20.536	182.952	50,6
MCC	16.900.000	24.271	2.680	57.719	70,5
DICOPT	17.500.000	17.935	2.104	26.279	123,3

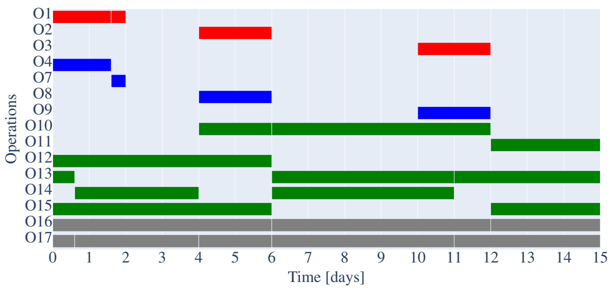

Figure 3: Gantt chart of the solution applying piecewise approximation by planes.

5. Conclusion

We presented a strategy for handling the product of two continuous variables based on piecewise linear approximation using planes. The main motivation for developing this strategy was to address the nonlinear constraint present in crude operations scheduling problems due to tank blending. We solved a case study based on an example from the literature and compared it with other strategies. Despite the PAP approach resulting in a larger model than the alternatives, it achieved the shortest solution time and obtained the highest objective function value alongside DICOPT. Future work will focus on refining the strategy formulation to reduce the number of variables and constraints involved.

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References

Castro, P. M., Grossmann, I. E., 10 2014. Global optimal scheduling of crude oil blending operations with rtn continuous-time and multiparametric disaggregation. Industrial and Engineering Chemistry Research 53, 15127–15145.
DOI: `10.1021/ie503002k`

Cerdá, J., Pautasso, P. C., Cafaro, D. C., 2015. Efficient approach for scheduling crude oil operations in marine- access refineries. Industrial and Engineering Chemistry Research 54.
DOI: `10.1021/acs.iecr.5b01461`

de Assis, L. S., Camponogara, E., Zimberg, B., Ferreira, E., Grossmann, I. E., 2017. A piecewise mccormick relaxation-based strategy for scheduling operations in a crude oil terminal. Computers and Chemical Engineering 106, 309–321.
DOI: `10.1016/j.compchemeng.2017.06.012`

Garcia-Verdier, T. G., Gutierrez, G., Méndez, C. A., Palacín, C. G., de Prada, C., 2024. Optimization of crude oil operations scheduling by applying a two-stage stochastic programming approach with risk management. Journal of Process Control 133, 103142.
DOI: `https://doi.org/10.1016/j.jprocont.2023.103142`

García-Verdier, T. G., Gutiérrez, G., Méndez, C., Palacín, C. G., de Prada, C., 1 2022. Minimizing risk in the scheduling of crudes in an oil refinery. IFAC-PapersOnLine 55, 809–814.
DOI: `10.1016/J.IFACOL.2022.07.544`

Lee, H., Pinto, J. M., Grossmann, I. E., Park, S., 1996. Mixed-integer linear programming model for refinery short-term scheduling of crude oil unloading with inventory management. Industrial and Engineering Chemistry Research 35.
DOI: `10.1021/ie950519h`

Mouret, S., Grossmann, I. E., Pestiaux, P., 9 2009. A novel priority-slot based continuous-time formulation for crude-oil scheduling problems. Industrial and Engineering Chemistry Research 48, 8515–8528.
DOI: `10.1021/ie8019592`

Yang, H., Bernal, D. E., Franzoi, R. E., Engineer, F. G., Kwon, K., Lee, S., Grossmann, I. E., 7 2020. Integration of crude-oil scheduling and refinery planning by lagrangean decomposition. Computers and Chemical Engineering 138.
DOI: `10.1016/j.compchemeng.2020.106812`